MATH 54 – MIDTERM 2 STUDY GUIDE

PEYAM RYAN TABRIZIAN

Note: Midterm 2 is on **Friday, July 13th** in 4 Evans from 12:05 pm to 2 pm. It covers sections $1.7-1.9, 4.1-4.7, 5.4$ of the Linear Algebra book and sections $4.2-4.6, 6.1-6.2$ of the Differential Equations - book. There will be about 10-11 questions, roughly 6-7 LA questions and 3-4 DE questions!

Note: 1.3.4 means 'Problem 4 in section 1.3'

CHAPTER 4: VECTOR SPACES

- Show that a set is or is not a vector space:
  - By either showing that the $\mathbf{0}$-vector is not in it, or by showing it is not closed under addition or scalar multiplication (4.1.1, 4.1.3, 4.1.7, 4.1.15)
  - By showing it's a subspace of a given vector space (4.1.5, 4.1.8, 4.1.20, 4.1.21, 4.1.22)
  - By expressing it as the span of some vectors (4.1.9, 4.1.17)
  - By writing it as $Nul(A)$ for some matrix A
- Given a matrix A, find $Nul(A)$ (4.2.5)
- Determine whether a set of vectors is linearly dependent or independent (1.7.1, 1.7.5, 1.7.7, 1.7.11, 1.7.15, 1.7.17)
- Show that a set of vectors, polynomials, matrices, functions is linearly dependent or independent (4.3.33)
- Show that a set is (or is not) a basis for a given vector space (4.3.1, 4.3.5, 4.3.6)
- Given a set of vectors, extract a basis for the span of those vectors (just form the matrix of those vectors, row-reduce, and find $Col(A)$, 4.3.15, 4.3.11)
- Find a basis for an abstract vector space (for example, find a basis for the set of 2×2 symmetric matrices, see practice midterm)
- Remember the trick of thinking of polynomials (or general vectors) as list of numbers! For example $1 - t + 2t^2$ is the same as $\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$ (4.3.34, 4.3.27)
- Find the dimension of a vector space (4.5.1, 4.5.3, 4.5.9, 4.5.10)
- Remember the trick that a set of 3 linearly independent vectors in a 3-dimensional vector space is a basis, etc.
- Given $[\mathbf{x}]_{\mathcal{B}}$, find $\mathbf{x}$, and vice-versa (4.4.1, 4.4.3, 4.4.5, 4.4.7, 4.4.11)
- Find the change-of-coordinates matrix from $\mathcal{B}$ to the standard basis in $\mathbb{R}^n$ (4.4.9)
- Given a matrix A, find a basis for $Nul(A)$, $Col(A)$, $Row(A)$, and also find $Rank(A)$ (4.6.1, 4.6.3, 4.6.4)
- Use the rank-nullity theorem to find $Rank(A)$ etc. (4.6.7, 4.6.9, 4.6.11, 4.6.13, 4.6.15)

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- Find the change-of-coordinates matrix from $\mathcal{B}$ to $\mathcal{C}$ (4.7.7, 4.7.9. always remember to evaluate the old vectors in $\mathcal{B}$ with respect to the new and cool basis $\mathcal{C}$)
- Use the change-of-coordinates matrix to find $[\mathbf{x}]_{\mathcal{C}}$ given $[\mathbf{x}]_{\mathcal{B}}$ and vice-versa (see examples covered in lecture)

LINEAR TRANSFORMATIONS

- Show that T is a linear transformation or not (1.8.32, 1.8.33, 5.4.5(b), 5.4.9(b), 5.4.10(a), or, for example, show $T(y) = y'' + 2y' - y$ is linear)
- Find the matrix of a given linear transformation T from $\mathbb{R}^n$ to $\mathbb{R}^m$ (1.9.1, 1.9.3, 1.9.5, 1.9.8, 1.9.10, 1.9.11, 1.9.17)
- Find the matrix of a linear transformation from an abstract vector space V to an abstract vector space W. Know how to do this with polynomials, functions, and matrices! (5.4.5(c), 5.4.7, 5.4.9(c))
- Remember the definitions of one-to-one and onto. Show that a linear transformation is one-to-one or not (Is the L.T. in 1.8.9 one-to-one?)
- Given a linear transformation T, find $Ker(T)$ (1.8.9, 4.2.32, or: what is the kernel of $T(y) = y'' - 5y' + 6y$)

CHAPTER 4: LINEAR SECOND-ORDER EQUATIONS

- Find the general solution to a second-order differential equation, possibly including complex roots, repeated roots, or initial conditions (4.2.1, 4.2.18, 4.3.1, 4.3.3, 4.3.24, but really, the other problems are good too!)
- Determine if two functions are linearly independent or linearly dependent (4.2.27, 4.2.28)
- Solve equations using undetermined coefficients (4.4.9, 4.4.11, 4.4.13, 4.4.15, 4.5.28, 4.2.30, I will not ask you for anything *too* complicated where one of the roots of the auxiliary equation coincides with the right-hand-side, just remember the basic stuff)
- Solve equations using variation of parameters (4.6.1, 4.6.3, 4.6.12, no need to remember any fancy integrals!)

CHAPTER 6: THEORY OF HIGHER-ORDER LINEAR DIFFERENTIAL EQUATIONS

- Find the largest interval on which a given differential equation has a unique solution (6.1.1, 6.1.3, 6.1.5)
- Use the Wronskian to show that a set of functions is linearly independent (6.1.7, 6.1.11, 6.1.13, 6.1.25)
- Find the general solution of a higher-order differential equation, possibly including initial conditions (6.2.1, 6.2.5, 6.2.9, 6.2.15, 6.2.17, 6.2.19, remember how to use the rational roots theorem!)

TRUE/FALSE EXTRAVAGANZA

Check out the following set of T/F questions (solutions are in the HW hints, but beware, there might be mistakes, e-mail me whenever something seems to be wrong): 1.8.21, 1.9.23, 4.1.24, 4.2.25, 4.2.26, 4.3.21, 4.3.22, 4.4.15, 4.4.16, 4.5.19, 4.5.20, 4.6.17, 4.6.18, 4.7.11, 4.7.12.

Note: There will be **NO** T/F questions about differential equations! However, there will be 5 T/F questions without justifications, and **4** T/F questions with justifications. They will *all* be linear algebra questions!

CONCEPTS

Here are a couple of concepts we learned so far. You **don't** have to memorize the definitions, just have a rough idea of what those things are

- Vector space, Subspace
- Linear (in)dependence, Span, Basis
- Dimension
- Nullspace, Row Space, Column Space, Rank
- Rank-Nullity Theorem
- Coordinates with respect to $\mathcal{B}$
- Change-of-coordinates matrix from $\mathcal{B}$ to $\mathcal{C}$, from $\mathcal{B}$ to the standard basis of $\mathbb{R}^n$
- Linear Transformation, the matrix of a linear transformation
- $Ker(T)$, $Ran(T)$, one-to-one, onto
- **The 3 miracles of linear algebra:**
  - Every vector space is like $\mathbb{R}^n$
  - Every linear transformation is a matrix, and every matrix is a linear transformation
  - If $dim(V) = dim(W) = n$ and T is one-to-one, then T is onto, and vice-versa
- Linear independence of functions, Wronskian matrix, Wronskian determinant
- Fundamental solution set
- Auxiliary polynomial